A modified TRiSK scheme to address instabilities found in MPAS-Ocean

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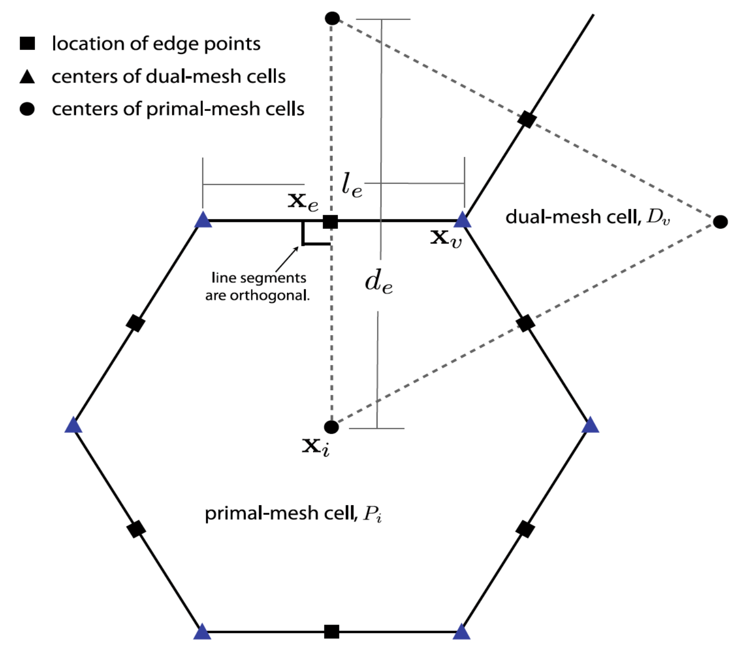
In MPAS-O the horizontal discretization used is the TRiSK scheme, a C-grid, finite-volume method applied to Spherical Centroidal Voronoi Tessellations (SCVTs) where the mass, tracers, pressure and kinetic energy are defined at centers of the convex polygons and the normal component of velocity is located at cell edges.



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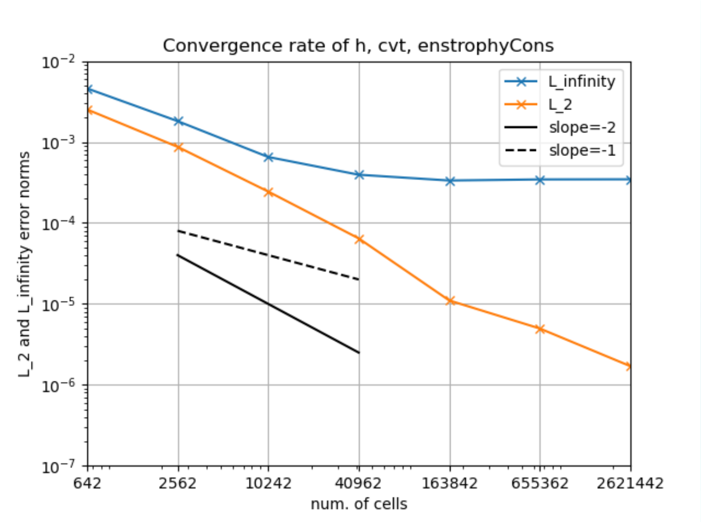
TRiSK has many desirable mimetic properties, it shows second order accuracy in the mean error norm (L2) in most shallow water test cases, but it fails to converge in the maximum norm.

Problem: this lack of convergence in the maximun norm can be carried to the 3D model causing potential instabilities.



Test 2 from Williamson et al., 1992

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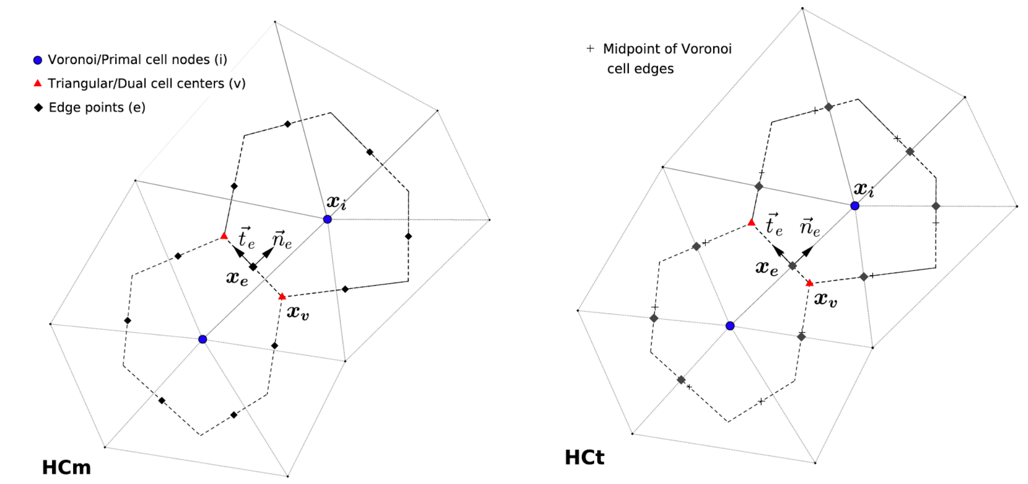
Strategy: based on previous analyses of Peixoto [1], we investigate a modification to the TRiSK scheme which consists of positioning the velocities on midpoints of the edges of the Voronoi cells, instead of the midpoint of the dual triangle edges.



HCm: new discretization
HCt: TRiSK

[1]: Peixoto, Pedro S. "Accuracy analysis of mimetic finite volume operators on geodesic grids and a consistent alternative." Journal of Computational Physics 310 (2016): 127-160.

E3SM Energy Exascale Earth System Model
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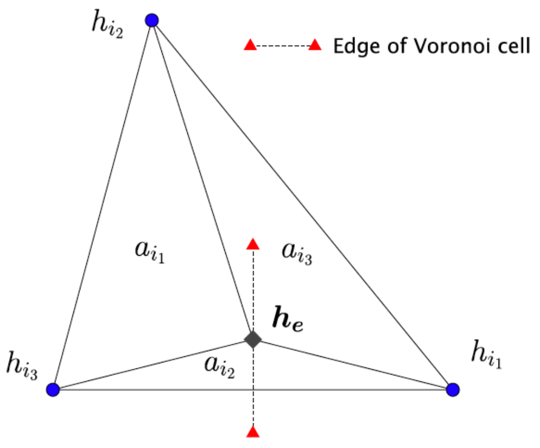
Interpolation of height

On a HCt grid: $h_e = (h_i + h_j)/2$
with i and j indexes of the Voronoi cells that share the edge.

On a HCm grid: $h_e = \sum_i \lambda_i(x_e)\, h_i$,
with the weights given by $\lambda_i = a_i(x_e)/A_k$
where $a_i(x_e)$ is the area of the triangle formed by the two vertices of the triangle opposite to i and the interpolation point for h_e, and A_k is the area of the triangle T_k.
This is called a *barycentric coordinate interpolation*.



Barycentric Coordinate Areas

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Kinetic energy

On a HCt grid: $K_i = \frac{1}{4A_i} \sum_e l_e d_e u_e^2$
where *e* varies within the edges of the Voronoi cell *i*, A_i is the Voronoi cell area and d_e and l_e are defined as the triangle and Voronoi edge lengths, respectively.

On a HCm grid: $K_i = \frac{1}{2} \overrightarrow{u_i} \cdot \overrightarrow{u_i}$, with $\overrightarrow{u_i} = \frac{1}{A_i} \sum_e n_{ei} (\overrightarrow{x_e} - \overrightarrow{x_i}) u_e l_e$
where x_e is the midpoint of the Voronoi cell edge, x_i is the center of the Voronoi cell and n_{ei} is a sign correction for the normal component.

Note: The expression for an HCm grid will in general NOT simplify to the energy conserving K_i of TRiSK.

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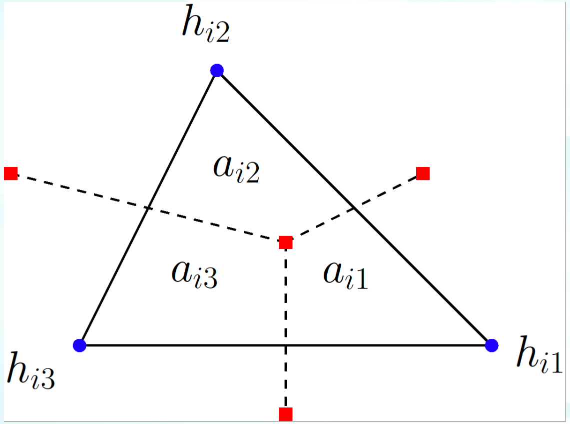
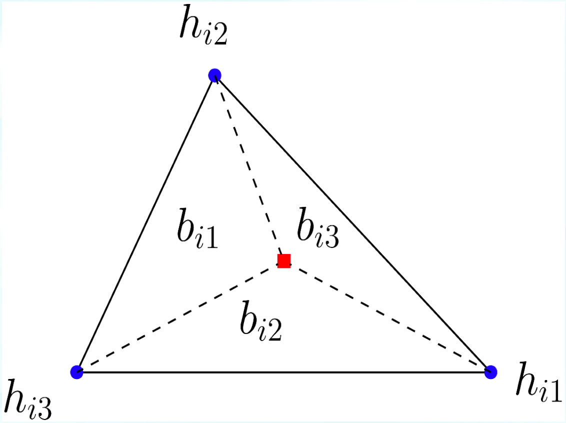
Potential vorticity

On a HCt grid: $h_v = \frac{1}{A_k} \sum_i a_{iv} h_i$



On a HCm grid: $h_v = \frac{1}{A_k} \sum_i b_{iv} h_i$



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Coriolis term

On a <u>HCt grid</u>: $w_{ee'} = \frac{c_{ee'}\, l_{e'}}{d_e}\left(\frac{1}{2} - \sum_v \frac{a_{iv}}{A_i}\right) n_{e'i}$

where a_{iv} is the overlapping area between the Voronoi cell and the dual triangular cell, the sum is within the vertices v between edge e and e', and c'_{ee} is a sign correction relative to the orientation of the tangent vector at e.

On a <u>HCm grid</u>: $w_{ee'} = \frac{1}{2}\frac{l_{e'}}{A_i} n_{e'i}(\overrightarrow{x_{e'}} - \overrightarrow{x_i}) \cdot \overrightarrow{t_e}$

where $\overrightarrow{t_e}$ unit vector tangent to the edge e.

Note: on a HCm grid the orthogonality property, $n_{e'i}(\overrightarrow{x_{e'}} - \overrightarrow{x_i}) = (d_{e'}\overrightarrow{n_{e'}})/2$, holds only approximately, and this may have a (small) impact on energy conservation.

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Progress made so far

We have implemented the changes to the horizontal divergence (specifically to h_e), kinetic energy, potential vorticity (specifically to h_v) and Coriolis term. We are now testing the new scheme on shallow water test cases to check that a first order accuracy is met.

Future work

Investigate the performance of this scheme applied to the full 3D ocean model to determine if it provides a good trade off in losing some properties of TRiSK in order to obtain first order accuracy and more stability.