A Suite of Verification Exercises for the Barotropic Solver of Ocean Models

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Motivation

- The development of any numerical ocean model warrants a suite of verification exercises to ensure if we are solving the equations right.

- This motivated me to design a set of shallow water test cases including dispersive and non-dispersive geophysical waves, barotropic tide etc. for testing the implementation of various terms in the barotropic equations of motion.

- Even though these verification exercises have been performed with the United States Department of Energy's Model for Prediction Across Scales – Ocean (MPAS-O), an unstructured ocean model with variable resolution capability, they can be run with any ocean model, structured or unstructured, and adopting any spatial and temporal discretization.

- From a pedagogical point of view, the visualization of these geophysical phenomena in addition to their standard mathematical analysis can help graduate students in atmospheric and oceanic sciences with a better understanding of the fundamental concepts, and a higher level of appreciation for the subject matter.

List of Test Cases

Geophysical Waves and Barotropic Tide

- Non-Dispersive Coastal Kelvin Wave
- Low Frequency Dispersive Planetary Rossby Wave
- Low Frequency Dispersive Topographic Rossby Wave
- High Frequency Dispersive Inertia Gravity Wave
- Non-Dispersive Equatorial Kelvin Wave
- Dispersive Equatorial Yanai Wave
- Low Frequency Dispersive Equatorial Rossby Wave
- High Frequency Dispersive Equatorial Inertia Gravity Wave
- Barotropic Tide

Standard Mathematical Test Cases

- Diffusion Equation
- Viscous Burger's Equation
- Non-linear Manufactured Solution in the Presence of Complex Bathymetry

List of Time-Stepping Algorithms

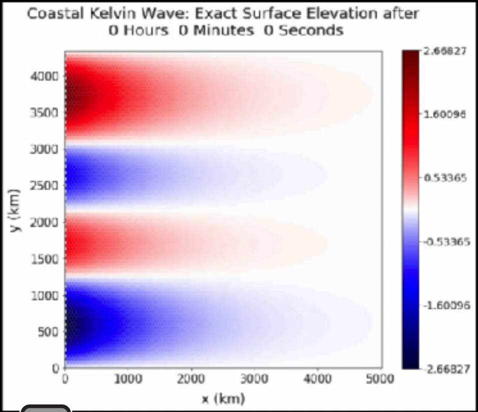
Standard Mathematical Time-Stepping Algorithms

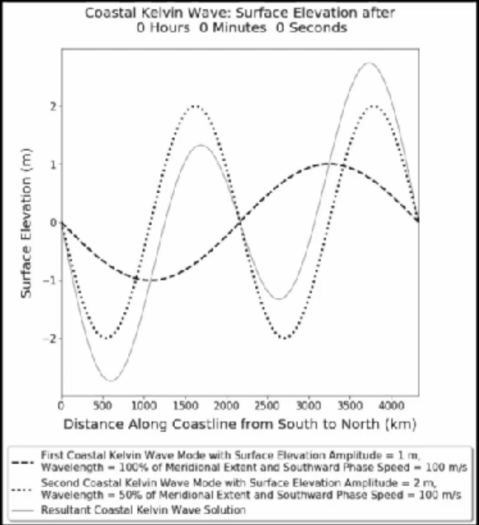
- Forward Backward or Implicit Euler
- Explicit Midpoint Method, a Form of Runga-Kutta Second Order
- Williamson's Low Storage Runga-Kutta Third Order
- Low Storage Runga-Kutta Fourth Order
- Adams Bashforth Second Order
- Adams Bashforth Third Order
- Adams Bashforth Fourth Order

Time-Stepping Algorithms Popular in Ocean Modeling

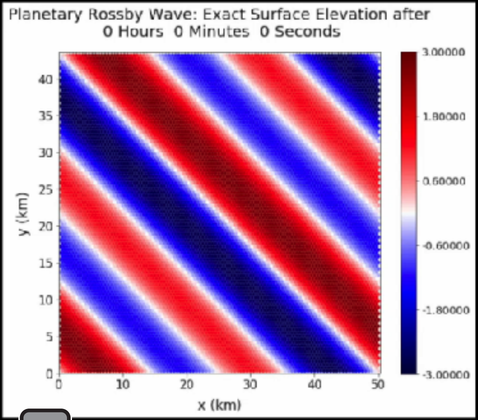
- Leapfrog Trapezoidal
- Leapfrog Adams Moulton
- Forward Backward with RK2 Feedback
- Generalized Forward Backward with AB2 - AM3 Step
- Generalized Forward Backward with AB3 - AM4 Step

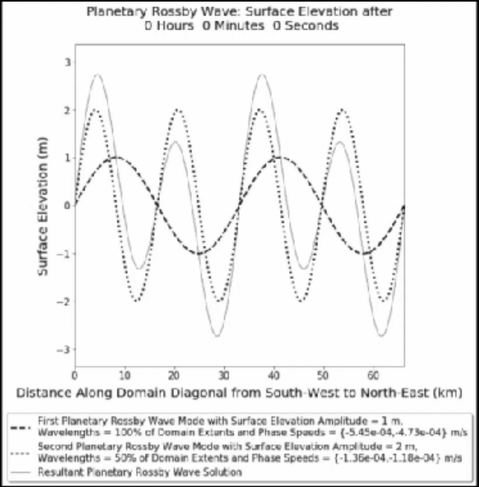
Idealized Tests: Coastal Kelvin Wave





Idealized Tests: Low Frequency Planetary Rossby Wave





Ongoing and Future Work

- Performing a thorough mathematical analysis of the order of convergence of hyperbolic and parabolic, linear and non-linear partial differential equations with spatial and temporal refinement

- Running convergence tests in space and time for each of the idealized test cases using a subset of the time-stepping algorithms and verifying the desired order of convergence

- Designing numerical experiments involving stratification i.e. multiple layers in the vertical direction, which would enable us to test both the barotropic and baroclinic time-stepping algorithms separately, as well as the barotropic-baroclinic splitting of the primitive equations

- Finalizing on the 'optimum' algorithms based on our experience with the idealized test cases, and employing them to perform global MPAS-O simulations

Related Publications in Progress

- On the Spatial and Temporal Convergence of Linear Partial Differential Equations

- On the Spatial and Temporal Convergence of Non-Linear Partial Differential Equations

- A Suite of Verification Exercises for the Barotropic Solver of Unstructured Ocean Models Part 1: Testing the Implementation of Linear Terms with Constant or Variable Coefficients

- A Suite of Verification Exercises for the Barotropic Solver of Unstructured Ocean Models Part 2: Testing the Implementation of Non-Linear Terms and Complex Bathymetry